CONTEREXAMPLES BY POSSIBLE CONJUNCTION

Premises provide the strongest support for their conclusion whenever it is impossible for all the premises to be true AND the conclusion false. In such cases the premises are (jointly) sufficient for the truth of their intended conclusion: necessarily, if they are all true, their conclusion is true, in other words, the inference is valid. A successful counterexample against the strongest support, is a description of any possible situation that makes us understand how it is possible for all the premises to be true AND their conclusion false. A successful counterexample proves that the premises are not jointly sufficient for their conclusion. Let us explore the logic of these counterexamples by inventing some against the following argument:

(A) (1) You are all intelligent. So, (C) each one of you will pass the course.

To determine whether premise A(1) provides the strongest support for conclusion A(C), we must answer the following questions, “Is A(1) sufficient or adequate for A(C)? Is it possible for A(1) to be true and for A(C) to be false?” Note that we are not asking whether premise A(1) is in fact true, or whether the conclusion is in fact false, and we are NOT trying to show that the premise is true and that the conclusion is false. We are asking ourselves, “IF the premise were true, could the conclusion be false?”

You already have an intuitive grasp of constructing counterexamples against the strongest support of premises. For most of you would correctly respond to argument (A) in one of the following succinct ways:
(a) Not necessarily! What if the teacher is not fair in his/her grading?
(b) Not necessarily! It is possible that the teacher is not fair in his/her grading.
(c) Not necessarily! The teacher could be unfair in his/her grading.

Your use of “Not necessarily” indicates that you understand that the conclusion does not necessarily follow from the given premise. Your use of “What if”, or “It is possible”, or “could” indicates that you understand that the statement showing that the given reason is not sufficient for its conclusion is only presented as a possibility: it is not asserted as actually true. I will name the statement(s) typically introduced by “what if” in a counterexample against the sufficiency of premises, the what-if-statement(s).

We will look at a number of counterexamples against argument (A) that successfully show that its premise is not sufficient for its conclusion, and then identify the main characteristics of these counterexamples. Now consider the complete counterexample:

CE1 It is possible that:
(1) You are all intelligent. [All the given premises are always granted, assumed true] AND
What-if-statement: The teacher is unfair in his/her grading. AND
not-(C): It is false that each one of you will pass the course.[The conclusion is always negated].

I name this kind of counterexample a counterexample by possible conjunction because it is simply a possible conjunction of propositions. (These counterexamples are different from counterexamples by analogy, which we will examine later in the course.) Such a counterexample, when it is successful, is any possible situation that makes us understand how it is possible for all the given premises to be true and the conclusion false.
Implicit Assumptions

Successful counterexamples against the strongest support of premises not only prove that all the given premises are not jointly sufficient, they also identify unstated (unexpressed, hidden, missing, implicit, tacit, suppressed) assumptions. In this case CE1 helps us to identify the hidden assumption that the teacher is not unfair, in other words, the teacher is fair. The unexpressed assumption is the contradictory of the what-if-statement in the counterexample. If we then show that this assumption is in fact false, we show that the argument rests on a false assumption. The evaluation of the truth or acceptability of explicit and implicit premises is an important part in the evaluation of an argument. But my purpose here is to focus on one way of evaluating the strongest support of all the given reasons of an argument.

Having identified that missing assumption, we can reconstruct argument (A) as:

(A') (1) You are all intelligent.
(2) The teacher is fair.
So, (C) each one of you will pass the course.

Once we identify this implicit assumption, and insert it into the argument as an explicit premise, no counterexample using the what-if-statement, “The teacher is not fair” can be advanced against (A'). Such counterexamples are blocked because any counterexample against an argument must always grant all the given premises of that argument in order to show that all its premises are not jointly sufficient for its conclusion; but now that argument (A') uses the statement “The teacher is fair” as a premise, we must grant it, and so we cannot use “The teacher is not fair” to construct a counterexample by possible conjunction. Since a counterexample is blocked, the premises' support for the conclusion is stronger than in argument (A). The premises of (A') provide stronger support in the sense that the probability of IF (1&2) THEN (C) is greater than the probability of IF (1), THEN (C).

Even though the strength of the support for the conclusion has increased, the conclusion still does not necessarily follow from those two premises because of the next counterexample by possible conjunction against the reconstructed argument (A'):

CE2: It is possible that:
(1) You are all intelligent. AND
(2) The teacher is fair. AND
What-if-statement: Some students do not do the required work. AND
not-(C): It is not the case that each one of you will pass the course.

Note again how the what-if-statements makes us understand how it is possible for all the given premises to be true and the conclusion false, and consequently, this counterexample proves that premises A'(1) and A'(2) are not jointly sufficient for the truth of A'(C).

This successful counterexample helps us to identify a further implicit assumption: the negation or contradictory of the what-if-statement: All students do the required work. So we can reconstruct argument A' as follows:

(A'') (1) You are all intelligent.
(2) The teacher is fair.
(3) All students do the required work.
So, (C) each one of you will pass the course.
This reconstruction blocks counterexamples such as CE2.

The support of the premises in argument (A") is stronger than the support of the premises in either arguments (A') or (A) because (A") blocks the counterexamples to which arguments (A') and (A) are vulnerable. A different way of expressing the same idea is that the probability of IF (1&2&3), THEN (C) is greater than either the probability of IF (1&2) THEN (C), or the probability of IF (1), THEN (C). However, despite this increase in premise support, the three premises of (A") are still not jointly sufficient to establish the truth of the intended conclusion. We can prove this with the next counterexample by possible conjunction:

CE3 It is possible that:
(1) You are all intelligent. AND
(2) The teacher is fair. AND
(3) All the students do the required work. AND
What-if-statement: some students do bad work. AND
not-(C): It is not the case that each one of you will pass the course.

Again the what-if-statement makes us understand how it is possible for all the given premises to be true and the conclusion false. By negating this what-if-statement we identify a further hidden assumption: No student does bad work. The insertion of this assumption into a reconstructed (A") gives us:

(A"') (1) You are all intelligent.
(2) The teacher is fair.
(3) All students do the required work.
(4) No student does bad work.
So, (C) each one of you will pass the course.

This reconstructed argument now blocks counterexample CE3. The support of the premises in (A"') for their conclusion is stronger than the support of the premises in arguments (A), (A'), and (A") for their respective conclusions.

Despite the increased support, the conclusion still does not necessarily follow:

CE4: It is possible that:
(1) You are all intelligent. AND
(2) The teacher is fair. AND
(3) All students do the required work. AND
(4) No student does bad work. AND
What-if-statement: An extraterrestrial abducts a student before s/he can do the required final exam. AND
not-(C): It is not the case that each one of you will pass the course.

Again the what-if-statement makes us understand how it is possible for all the given premises to be true and the conclusion false. But note that this counterexample is much less likely than the preceding ones. Though all counterexamples by possible conjunction are equally logically possible, they are not equally probable. And despite the very extreme unlikelihood of this far-
fetched counterexample, we have again identified an (understandably) unexpressed assumption: no extraterrestrial abducts a student before s/he can do the required final exam. A reconstruction of A” results in an argument whose premises provide even stronger support than the respective premises of all the preceding arguments.

(A’’) (1) You are all intelligent.
(2) The teacher is fair.
(3) All students do the required work.
(4) No student does bad work.
(5) No extraterrestrial abducts a student before s/he can do the required final exam.
So, (C) each one of you will pass the course.

Though the premises in argument A”” provide stronger support than the premises in arguments A”, A”, A’, and A, the five premises of A”” are still not sufficient to establish the truth of the intended conclusion. Can you invent further counterexamples?

When do we stop inventing counterexamples by possible conjunction?
(1) If the author of the argument intended the conclusion to follow necessarily from its premises, and there is no need to assess the degree of support that the premises bring to the conclusion, we stop at the first counterexample. From a strictly logical point of view, any counterexample, no matter how wildly imaginative and unlikely, proves that the conclusion does not necessarily follow. However, very unlikely counterexamples (e.g. CE5) will not always persuade one’s audience.

(2) We stop when a counterexample helps us to identify a hidden assumption that we can then show to be false, AND the argument needs that assumption to provide the intended degree of support.

(3) There is NO need to invent a counterexample when all the premises are actually true and the conclusion actually false. Such a situation – by itself – suffices to show that the premises are not sufficient for the conclusion.

The next condition is VERY IMPORTANT because it helps us to use these same counterexamples to estimate the degree of support of premises that is less than conclusive, and thus to assess many non-deductive arguments. If we do not learn this additional use of these counterexamples, we will tend to hastily reject arguments after the first counterexample, and consequently reject many strong arguments. We must learn that the support of premises can be strong even if it is not conclusive.

(4) If the author of the argument does not intend the conclusion to follow logically, in other words, if the premises are not intended to be sufficient for the truth of the conclusion, but only to render the conclusion probable, we stop inventing counterexamples in the following cases:

(a) The estimated combined probability of all the counterexamples we have invented is sufficiently high to show that the given premises (when assumed to be true) fail to render to conclusion probable to the intended degree.
Consider again the argument “You are all intelligent, so each one of you will pass the course”, and the four counterexamples against it: What if the teacher is unfair in his/her grading; what if some students do not do the required work; what if some students do bad work; what if an extraterrestrial abducts a student before s/he can do the required final exam. If, for example, one estimates that the combined four counterexamples against argument has a high probability, then, relative to one’s knowledge and imagination, the support of the premise is low. In other words, if the premise that you are all intelligent were true, the probability of the conclusion that each one of you will pass is low. So, if the author intended the conclusion to be very probable or even just probable, then these combined counterexamples would show that the author is mistaken. We can then stop inventing counterexamples.

(b) The estimated combined probability of all the counterexamples we have invented is not sufficiently high to show that the given premises (when assumed to be true) fail to render the conclusion probable to the intended degree, AND we have sufficient reason to believe that our attempt to invent additional counterexamples will be similarly inadequate. For example, if some premises are intended to render their conclusion very probable, and the only counterexamples we can invent are extremely improbable, then, relative to our level of knowledge and imagination, the given premises render the conclusion very probable.

Characteristics of a successful counterexample by possible conjunction:
(a) It is any possible situation, any possible conjunction of statements, any finite group of consistent statements (i.e. statements that can all be true together, at the same time),
(b) in which all the given premises of an argument are granted, they are all assumed to be true, and
(c) the intended conclusion is negated, is assumed false.
(d) The other statements (those that I have labeled “what-if-statements”) conjoined to the given premises and the negated conclusion make us understand how it is possible for all the given premises to be true and the conclusion false.
(e) The negation of the what-if-statement identifies a hidden (unexpressed, unstated, missing, tacit, implicit, suppressed) assumption of the argument.

A counterexample by possible conjunction by itself is not an argument. It is a finite set of consistent propositions, but we can use it to prove that the given premises of an argument are not jointly sufficient to establish the truth of its conclusion. For example, we can use the first counterexample CE1 to prove that premise A(1) by itself is not sufficient to establish the truth of the conclusion A(C). Our ability to create counterexamples depends on our imagination and on our knowledge of the issue in question. If we fail to invent better counterexamples (or invent probable ones), it could be because we have not properly used our imagination, or because we lack relevant knowledge. So, if we are unable to invent a counterexample, it does not logically follow that the argument is in fact valid. However, we can infer in such cases is that, relative to our current knowledge and imagination, the premises are sufficient for their conclusion, in other words, relative to our current knowledge and imagination, the argument is valid.
Don’t forget to distinguish the different uses of these counterexamples. (a) The above examples illustrate how they are used to show that reasons in arguments are not sufficient for a conclusion. In such cases we are simply proving that certain statements are not sufficient for the truth of some other statement. (b) We can use similar counterexamples to prove that propositions are not sufficient for some other proposition in a conditional statement. For example, given, “If s/he is a professor, then s/he is psychotic”, the proposition “S/he is a professor” is shown not to be sufficient for the proposition “S/he is psychotic” by either imagining or pointing to some real (there are at least a few, you just need to look hard) cases where a professor is not psychotic. (c) We similarly use counterexamples to prove that concepts are not sufficient for other concepts. In “All professors are psychotic” the concept of being a professor is assumed to be sufficient for the concept of being psychotic. (d) We can use the same counterexamples to criticize the alleged sufficiency of causal factors. In “The fear made him/her flee” the fear is assumed to be sufficient to cause the fleeing. A counterexample would consist of imagining or identifying actual situations where the individual experiences the same fear (same intensity, duration, frequency), but does not flee.

Different but logically equivalent questions to raise when assessing the strongest support of premises:
1) Is it impossible for all the premises to be true and the conclusion to be false? Yes: valid. No: invalid.
2) Is it inconsistent to assume all the premises true and the conclusion false? Yes: valid. No: invalid.
3) Are all the premises jointly sufficient/adequate for the truth of the conclusion? Yes: valid. No: invalid.
4) Necessarily, if all the premises are true, is the conclusion true? Yes: valid. No: invalid.
5) Does the truth of all the premises necessitate the truth of the conclusion? Yes: valid. No: invalid.

Do not forget that the unstated (missing, unexpressed, hidden, suppressed, implicit, tacit) assumptions/premises identified by negating the what-if-statements in a successful counterexample by possible conjunction differ from tacit premises whose insertion into an argument render it valid. Such tacit premises make a conceptual connection between premises and a conclusion, for instance, in the argument “All profs are neurotic, so all profs are happy”, the kind of hidden assumption/premise that connects central concepts in the premise to central concepts in the conclusion is “All neurotics are happy”. Its insertion renders the argument valid. But the insertion of a negated what-if-statement does not render an argument valid.
1. ARGUMENT: You’re at least eighteen years old because you’re in your second year of college.

CE by possible conjunction. It is possible that:

| You’re in your second year of college. AND | What-if statement: | AND | It is false that you’re at least eighteen years old |

RECONSTRUCTED ARGUMENT
You’re in your second year of college.
NEGATION of the what-if statement: ________________________________
So, you’re at least eighteen years old.

2. ARGUMENT: She ate just a moment ago, so she does not need to eat again now.

CE by possible conjunction. It is possible that:

| She ate just a moment ago. AND | What-if statement: | AND | It is false that she does not need to eat again now. |

RECONSTRUCTED ARGUMENT
She just ate a moment ago.
NEGATION of the what-if statement: ________________________________
Hence, she does not need to eat again now.

3. ARGUMENT: You will see much better now, for you now have new glasses.

CE by possible conjunction. It is possible that:

| You now have new glasses. AND | What-if statement: | AND | It is false that you will see much better now. |

RECONSTRUCTED ARGUMENT
You now have new glasses.
NEGATION of the what-if statement: ________________________________
All this leads me to believe that you will see much better now.

4. ARGUMENT: I can accurately describe his political views because I’m his wife.

CE by possible conjunction. It is possible that:

| I am his wife. AND | What-if statement: | AND | It is false that I can accurately describe his political views. |

RECONSTRUCTED ARGUMENT
I’m his wife.
NEGATION of the what-if statement: ________________________________
Consequently, I can accurately describe his political views.
5. ARGUMENT: The number of homeless people in our city is decreasing, **consequently** the city council must be doing something right.

CE by possible conjunction. **It is possible that:**

<table>
<thead>
<tr>
<th>The number of homeless people in our city is decreasing. <strong>AND</strong></th>
<th>What-if statement:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>It is false that</strong> city council must be doing something right.</td>
<td></td>
</tr>
</tbody>
</table>

**RECONSTRUCTED ARGUMENT**

The number of homeless people in our city is decreasing.

NEGATION of the what-if statement: ________________________________

**I can infer that** the city council must be doing something right.

6. ARGUMENT: All art is essentially revolutionary **for** all art essentially appeals to freedom.

CE by possible conjunction. **It is possible that:**

<table>
<thead>
<tr>
<th>All art essentially appeals to freedom. <strong>AND</strong></th>
<th>What-if statement:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>It is false that</strong> all art is essentially revolutionary.</td>
<td></td>
</tr>
</tbody>
</table>

**RECONSTRUCTED ARGUMENT**

All art essentially appeals to freedom.

NEGATION of the what-if statement: ________________________________

**It follows that** all art is essentially revolutionary.

7. ARGUMENT: Young people with Down syndrome are achieving great things by holding down full time jobs.

CE by possible conjunction: **It is possible that:**

<table>
<thead>
<tr>
<th>[Some young people with Down syndrome are] holding down full time jobs. <strong>AND</strong></th>
<th>What-if statement:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>It is false that</strong> [some] young people with Down syndrome are achieving great things.</td>
<td></td>
</tr>
</tbody>
</table>

**RECONSTRUCTED ARGUMENT**

[Some young people with Down syndrome are] holding down full time jobs.

NEGATION of the what-if statement: ________________________________

**Accordingly**, [some] young people with Down syndrome are achieving great things.

8. ARGUMENT: I can persuade any fanatically ascetic monk to take a drink of alcohol. **Thus**, I can persuade anyone to take a drink of alcohol.

CE by possible conjunction: **It is possible that:**

<table>
<thead>
<tr>
<th>I can persuade any fanatically ascetic monk to take a drink alcohol. <strong>AND</strong></th>
<th>What-if statement:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>It is false that</strong> I can persuade anyone to take a drink of alcohol.</td>
<td></td>
</tr>
</tbody>
</table>

**RECONSTRUCTED ARGUMENT**

I can persuade any fanatically ascetic monk to take a drink alcohol.

NEGATION of the what-if statement: ________________________________

**Given these facts**, I can persuade anyone to take a drink of alcohol.
What-if statements
1. started elementary school at an early age
   Took many college credits in high school.
   Skipped one or two grades
   Genius

2. still hungry
   ate small quantity, e.g. part of a seven course meal
   low blood sugar, and needs to eat to bring up the blood sugar

3. non-prescription glasses, e.g. sun glasses
   now the eyes need to adjust to the new prescription glasses
   wrong prescription
   caused by cataract, now rectified by glasses
   just a new frame
   blind, bought glasses to be trendy

4. separated, divorced
   husband has no interest in politics, has no views, not communicated to wife,
   they don’t discuss politics

5. kill
   they moved out,
   starved, died from exposure to the elements,
   other organization provided the help
   economy improved radically
   XX inaccurate counting of homeless XX this one is inconsistent with the premise

6. same style of music in which one appeals to the freedom of self-expression
   artist creating freely, but not breaking new artistic grounds

7. job not at all demanding, home-based job, the boss is extremely tolerant of incompetent work

8. someone has greater self-control or self-restraint than any monk