

TEST III

Spring 2011

MATH 115

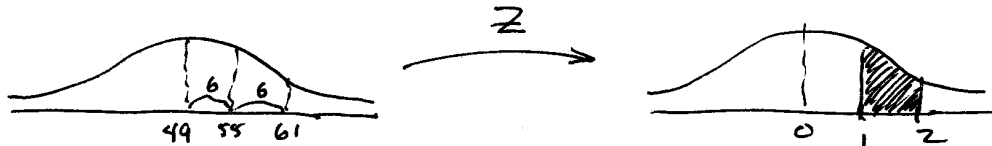
Chapters 6, 7 and 8

Name Key

There is no need for a Scantron on this test. For the multiple choice questions, you do not need to show your work. For the other questions, you should show your work, but please make it neat.

1. Dave drives to work each morning at about the same time. His commute time is normally distributed with a mean of 49 minutes and a standard deviation of 6 minutes. The percentage of time that his commute time lies between 55 and 61 minutes is equal to the area under the standard normal curve between 1 and 2 1 D

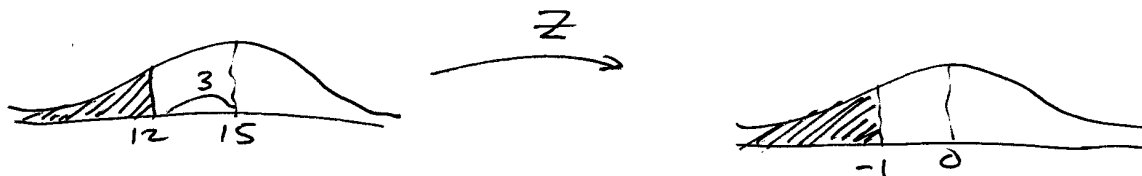
- A) 1.2, 2.5 B) 0.5, 1.5 C) 0, 1 D) 1, 2



2. The amount of time that customers wait in line during peak hours at popular fast food restaurant is normally distributed with a mean of 15 minutes and a standard deviation of 3 minutes. The percentage of time that the waiting time is less than 12 minutes is equal to the area under the standard normal curve that lies to the left of -1. 2 C

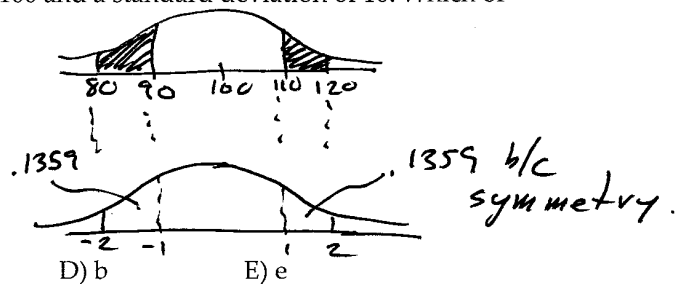
- A) left, 0.5 B) right, 1 C) left, -1 D) right, -1

$$z = \frac{12 - 15}{3} = \frac{-3}{3} = -1$$



3. The area under the standard normal curve between 1 and 2 is equal to 0.1359. Scores on a particular aptitude test are normally distributed with a mean of 100 and a standard deviation of 10. Which of the following are equal to 13.59%? 3 A

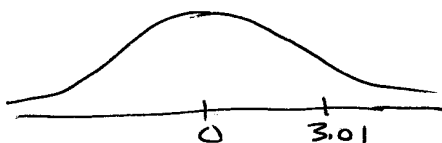
- a. The percentage of scores between 120 and 130
- b. The percentage of scores between 110 and 120
- c. The percentage of scores between 80 and 90
- d. The percentage of scores between 90 and 120
- e. The percentage of scores between 90 and 110



- A) b, c B) d C) a, b D) b E) e

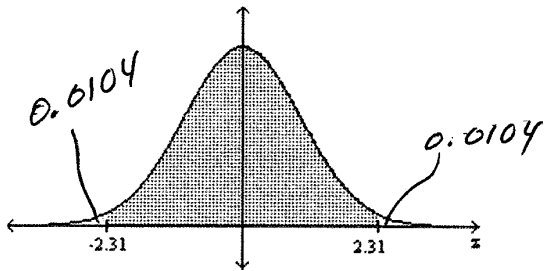
In problems 4 and 5, find the specified area under the standard normal curve.

4. The area that lies between 0 and 3.01 4 D
- A) 0.1217 B) 0.5013 C) 0.9987 D) 0.4987



$$\begin{aligned} \text{table}(3.01) &= 0.9987 \\ -\text{table}(0) &= -0.5 \\ \hline &0.4987 \end{aligned}$$

5. The unshaded area shown



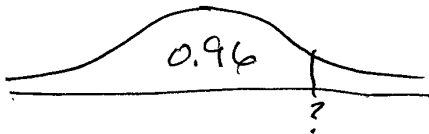
- A) 0.9896 B) 0.0104 C) 0.0208 D) 9688

$$\begin{aligned} \text{table}(-2.31) &= 0.0104 \\ &\times 2 \\ \hline &0.0208 \end{aligned}$$

5 C

6. Find the z-score for which the area under the standard normal curve to its left is 0.96

- A) 1.03 B) 1.75 C) 1.82 D) -1.38

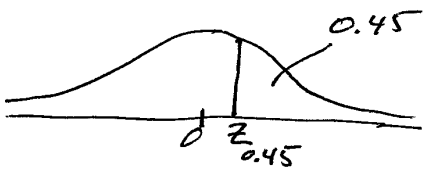


$$\begin{aligned} \text{table}^{-1}(0.96) &= 1.75 \\ \text{i.e., table}(1.75) &= 0.96 \end{aligned}$$

6 B

7. Find $z_{0.45}$.

- A) 0.6736 B) -0.13 C) 0.13 D) 0.3264



$$\begin{aligned} \text{table}^{-1}(1-0.45) \\ &= \text{table}^{-1}(0.55) \approx 0.13 \\ \text{i.e., table}(0.13) &= 0.55 \end{aligned}$$

7 C

8. Which of the following statements concerning the standard normal curve is/are true (if any)?

- a. The area under the standard normal curve to the left of -3 is zero.
- b. The area under the standard normal curve between any two z-scores is greater than zero.
- c. The area under the standard normal curve between two z-scores will be negative if both z-scores are negative.
- d. The area under the standard normal curve to the left of any z-score is less than 1.

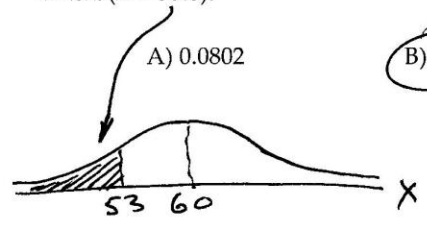
- A) a B) a, b C) a, c D) b, d

both b and d are correct

8 D

9. The variable X is normally distributed. The mean is $\mu = 60.0$ and the standard deviation is $\sigma = 4.0$. Find $P(X < 53.0)$.

9 B



$$z = \frac{53 - 60}{4} = \frac{-7}{4} = -1.75$$

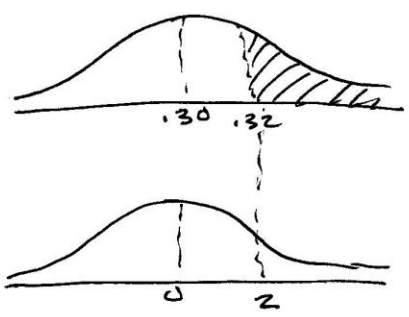
$$\text{table}(-1.75) = 0.0401$$



$$\therefore P(X < 53) = 0.0401$$

10. The diameters of bolts produced by a certain machine are normally distributed with a mean of 0.30 inches and a standard deviation of 0.01 inches. What percentage of bolts will have a diameter greater than 0.32 inches? Round your answer to two decimal places, e.g., 38.37%

10 2.28%



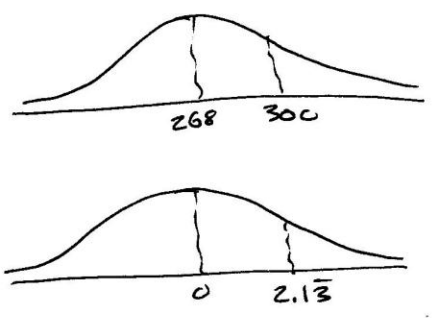
$$z = \frac{.32 - .30}{.01} = \frac{+.02}{+.01} = 2$$

$$1 - \text{table}(2) = .0228$$

$$\Rightarrow 2.28\%$$

11. The lengths of human pregnancies are normally distributed with a mean of 268 days and a standard deviation of 15 days. What is the probability that a pregnancy lasts at least 300 days? Answer to 4 decimal places.

11 0.0166

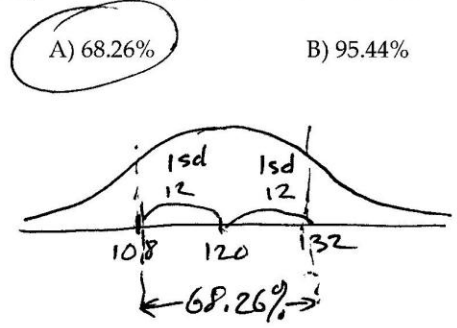


$$z = \frac{300 - 268}{15} = \frac{32}{15} = 2.1\bar{3}$$

$$1 - \text{table}(2.1\bar{3}) = 1 - 0.9834 = 0.0166$$

12. The systolic blood pressure of 18-year-old women is normally distributed with a mean of 120 mmHg and a standard deviation of 12 mmHg. What percentage of 18-year-old women have a systolic blood pressure between 108 mmHg and 132 mmHg?

12 A



from empirical rule
1 std dev \Rightarrow 68.26%

- A) 68.26% B) 95.44% C) 99.99% D) 99.74%

13. The table reports the GPA for each of five students in a statistics class.

13 C

Student	Maria	Alvin	Elvis	Ingrid	Rashad
GPA	3.52	3.65	3.66	3.92	3.95

For a random sample of size two, find the probability, expressed as a percent, that the sample mean will be within 0.1 of the population mean.

A) 30%

B) 20%

C) 70%

D) 80%

MA 3.585
 ME 3.59
 MI 3.72
 MR 3.785
 AE 3.655
 AI 3.785
 AR 3.80
 EI 3.79
 ER 3.805
 IR 3.935

$$\mu_{\bar{x}} = \mu_x = 3.74$$

$$P(\mu - 0.1 \leq \bar{X} \leq \mu + 0.1)$$

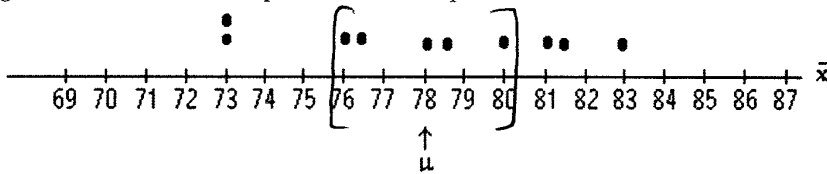
$$P(3.64 \leq \bar{X} \leq 3.84)$$

just count them.

7 of them. Hence $\frac{7}{10} = 0.7$ or 70%

14. The test scores of 5 students are under consideration. The following is the dotplot for the sampling distribution of the sample mean for samples of size 2.

14 B



Find the probability, expressed as a percent, that the sample mean will be within 2 points of the population mean.

A) 40%

B) 50%

C) 60%

D) 30%

just count dots from $\mu - 2$ to $\mu + 2$.
 there are 5 dots from 76 to 80
 hence 5, $\frac{5}{10} = 50\%$

15. The National Weather Service keeps records of rainfall in valleys. Records indicate that in a certain valley, the annual rainfall has a mean of 86 inches and a standard deviation of 10 inches. Suppose the rainfalls are sampled during randomly picked years, and \bar{x} is the mean amount of rain in these years. For samples of size 25, determine the mean and standard deviation of \bar{x} .

15 A

A) $\mu_{\bar{x}} = 86$; $\sigma_{\bar{x}} = 2$

B) $\mu_{\bar{x}} = 10$; $\sigma_{\bar{x}} = 86$

C) $\mu_{\bar{x}} = 2$; $\sigma_{\bar{x}} = 86$

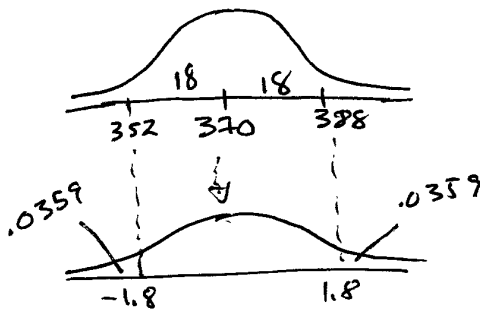
D) $\mu_{\bar{x}} = 86$; $\sigma_{\bar{x}} = 10$

$$\mu_{\bar{x}} = \mu = 86$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{25}} = \frac{10}{5} = 2$$

16. The monthly expenditures on food by single adults living in one neighborhood of Los Angeles are normally distributed with a mean of \$370 and a standard deviation of \$50. Determine the percentage of samples of size 25 that have a mean expenditure within \$18 of the population mean expenditure of \$370. Round your answer to two decimal places, e.g. 38.37%.

16 92.82%



$$n = 25$$

$$z = \frac{18}{\sigma/\sqrt{n}} = \frac{18}{50/\sqrt{25}} = \frac{18}{10} = 1.8$$

$$\text{table } (-1.8) = 0.0359$$

$$\frac{0.0359 \times 2}{0.0718}$$

$$1 - 0.0718 = 0.9282$$

17. For the population of one town, the number of siblings is a random variable whose relative frequency histogram has a reverse J-shape. Let \bar{x} denote the mean number of siblings for a random sample of size 30. For samples of size 30, which of the following statements concerning the sampling distribution of the mean is true?

17 B

- A) \bar{x} is normally distributed. B) \bar{x} is approximately normally distributed.
 C) The distribution of \bar{x} has a reverse J-shape. D) None of the above statements is true.

by Central Limit theorem, \bar{x} is approximately normally dist.

18. A long-distance phone company wishes to estimate the mean duration of long-distance calls originating in California. A random sample of 15 long-distance calls originating in California yields the following call durations, in minutes.

18 15

5 4 3 1 2
 34 29 40 24 12
 1 19 12 2 37

$$\sum x_i = 225$$

Use the data to obtain a point estimate of the mean call duration for all long-distance calls originating in California. Round your answer to one decimal place, e.g. 29.3 minutes.

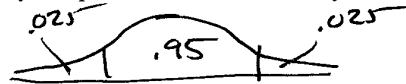
$$\bar{x} = \frac{\sum x_i}{n} = \frac{225}{15} = 15$$

19. 34 packages are randomly selected from packages received by a parcel service. The sample has a mean weight of 21.1 pounds. Assume that $\sigma = 2.4$ pounds. What is the 95% confidence interval for the true mean weight, μ , of all packages received by the parcel service? Round your answer to one decimal place, e.g., 38.7 pounds.

19 20.3
21.9

$$n = 34$$

$$\frac{\alpha}{2} = \frac{1 - 0.95}{2} = \frac{0.05}{2} = 0.025$$



$$\text{table}^{-1}(0.02) = -1.96 \quad \therefore z_{\alpha/2} = 1.96$$

$$CI = \bar{x} - z_{\alpha/2} \left(\frac{\sigma}{\sqrt{n}} \right) \text{ to } \bar{x} + z_{\alpha/2} \left(\frac{\sigma}{\sqrt{n}} \right)$$

$$= 21.1 - 1.96 \left(\frac{2.24}{\sqrt{34}} \right) \text{ to } 21.1 + 1.96 \left(\frac{2.24}{\sqrt{34}} \right)$$

$$= 20.3 \rightarrow 21.9$$

20. Based on a sample of 40 randomly selected years, a 90% confidence interval for the mean annual precipitation in one city is from 42.6 inches to 45.4 inches. Find the margin of error. Round your Answer to one decimal places, e.g. 8.9 inches.

20 1.4 in

$$CI = 42.6 - 45.4 = 2.8 \text{ inches}$$

$$\text{Margin of error} = \frac{1}{2}(2.8) = 1.4 \text{ inches}$$

21. Weights of women in one age group are normally distributed with a standard deviation σ of 13 lb. A researcher wishes to estimate the mean weight of all women in this age group. Find how large a sample must be drawn in order to be 90.8 percent confident that the sample mean will not differ from the population mean by more than 3.9 lb. find n.

21 32

~~CI = 42.6 - 45.4 = 2.8 inches~~

$$1 - \alpha = .908 \Rightarrow \frac{\alpha}{2} = \frac{.092}{2} = .046 \quad \text{table}^{-1}(.046) \Rightarrow Z_{\frac{\alpha}{2}} = 1.685$$

$$n > \left(\frac{Z_{\frac{\alpha}{2}} \sigma}{E} \right)^2 = \left(\frac{(1.685)(13)}{3.9} \right)^2 = 31.55 \Rightarrow n = 32$$

22. A confidence interval for a population mean has a margin of error of 3.3. If the sample mean is 51.2, obtain the confidence interval. Round your answer to two decimal places.

22 47.9 → 54.5

$$CI = 51.2 - 3.3 \text{ to } 51.2 + 3.3 \\ 47.9 \text{ to } 54.5$$

23. A laboratory tested twelve chicken eggs and found that the mean amount of cholesterol was 210 milligrams with $s = 18.1$ milligrams. Construct a 95% confidence interval for the true mean cholesterol content of all such eggs. Round your answer to one decimal place.

23 198.5 → 221.5

$$n = 12 \quad 1 - \alpha = .95 \quad \frac{\alpha}{2} = \frac{.05}{2} = .025 \quad \therefore t_{\frac{\alpha}{2}} = 2.201$$

$$CI = 210 \pm 2.201 \left(\frac{18.1}{\sqrt{12}} \right) = 210 \pm 11.5$$

$$CI = 198.5 \rightarrow 221.5$$

24. Which of the following statements regarding t-curves is/are true?

24 D

- a. The total area under a t-curve with 10 degrees of freedom is greater than the area under the standard normal curve.
- b. The t-curve with 10 degrees of freedom is flatter and wider than the standard normal curve.
- c. The t-curve with 10 degrees of freedom more closely resembles the standard normal curve than the t-curve with 20 degrees of freedom.

A) a

B) b, c

C) c

D) b

*a is not true because areas under both curves are 1.
b is true
c is not true, 20 df curve is closer to a normal curve than a 10 df curve
∴ D*

25. What day is the final exam on?

25 A

A) Last day of class, June 1st

Final Exam is required in this class.