Continuous spectra and blackbody radiation

• A blackbody is an idealized case of a hot, dense object.

• The figure shows the continuous spectrum produced by a blackbody at different temperatures.

• Max Planck provided the first explanation of blackbody radiation by assuming that atoms in the blackbody have evenly spaced energy levels, and emit photons by jumping from one energy level down to the next one.
Blackbody radiation derived through classical methods. The ultraviolet catastrophe: The Rayleigh–Jeans law does not explain the observed blackbody emission spectrum.

\[ I_\lambda(T) = \frac{2c k_B T}{\lambda^4} \]
Blackbody radiation derived using newly formed ideas of quantized energy. Planck’s theoretical result (continuous curve) and the experimental blackbody radiation curve (dots).

\[ I_\lambda(T) = \frac{2hc^2}{\lambda^5} \left( e^{\frac{hc}{\lambda k_B T}} - 1 \right)^{-1} \]
Continuous spectra and blackbody radiation

• The spectral emittance $I(\lambda)$ for radiation from a blackbody has a peak whose wavelength depends on temperature:

  ![Wien displacement law for a blackbody](image)

  \[ \lambda_m T = 2.90 \times 10^{-3} \text{ m} \cdot \text{K} \]

  Absolute temperature of blackbody

• We can obtain the Stefan–Boltzmann law for a blackbody by integrating $I(\lambda)$ over all wavelengths to find the total radiated intensity:

  \[
  I = \int_0^\infty I(\lambda) \, d\lambda = \frac{2\pi^5 k^4}{15c^2h^3} T^4 = \sigma T^4
  \]

  where $\sigma = 5.6704 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$ is the Stefan–Boltzmann constant.
A brass plate at room temperature (300 K) radiates 10 W of energy. If its temperature is raised to 600 K, it will radiate

A. 20 W
B. 40 W
C. 80 W
D. 160 W
A brass plate at room temperature (300 K) radiates 10 W of energy. If its temperature is raised to 600 K, the wavelength of maximum radiated intensity

A. Increases.
B. Decreases.
C. Remains the same.
D. Not enough information to tell.
Example 1 – A 2.0-cm-diameter metal sphere is glowing red, but a spectrum shows that its emission spectrum peaks at an infrared wavelength of 2.0 µm. How much power does the sphere radiate? Assume the metal sphere acts as a blackbody.
Around 1900, Phillip Lenard built an apparatus that produced an electric current when ultraviolet light was shining on the cathode.

This phenomenon is called the **photoelectric effect**.
Photoelectrons are emitted only if the light frequency $f$ exceeds a threshold frequency $f_0$.

- The value of the threshold frequency $f_0$ depends on the type of metal from which the cathode is made.

No matter how weak the light, there is a current if $f > f_0$.

No matter how intense the light, there is no current if $f < f_0$. 
- If the voltage between the cathode and anode is decreased and made negative, the current decreases.
- Current reaches zero when $\Delta V = -V_{\text{stop}}$, where $V_{\text{stop}}$ is the stopping potential.

![Graph showing characteristics of the photoelectric effect.](https://example.com/graph.png)
A minimum energy is needed to free an electron from a metal. This minimum energy is called the work function $E_0$ of the metal. Some deeper electrons may require more energy than $E_0$ to escape, but all will require at least $E_0$.

**TABLE 38.1** The work function for some of the elements

<table>
<thead>
<tr>
<th>Element</th>
<th>$E_0$ (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Potassium</td>
<td>2.30</td>
</tr>
<tr>
<td>Sodium</td>
<td>2.75</td>
</tr>
<tr>
<td>Aluminum</td>
<td>4.28</td>
</tr>
<tr>
<td>Tungsten</td>
<td>4.55</td>
</tr>
<tr>
<td>Copper</td>
<td>4.65</td>
</tr>
<tr>
<td>Iron</td>
<td>4.70</td>
</tr>
<tr>
<td>Gold</td>
<td>5.10</td>
</tr>
</tbody>
</table>
An electron with energy $E_{\text{elec}}$ inside a metal loses a minimum amount of energy $E_0$ as it escapes, so it emerges with maximum kinetic energy $K_{\text{max}} = E_{\text{elec}} - E_0$.

$\Delta V = 0$: The photoelectrons leave the cathode in all directions. Only a few reach the anode.
The Stopping Potential

- A positive anode attracts the photoelectrons.
- Once all electrons reach the anode, a further increase in $\Delta V$ does not cause any further increase in the current $I$.

$\Delta V > 0$: A positive anode attracts the photoelectrons to the anode.
The current decreases as the anode voltage becomes increasingly negative until, at the stopping potential, all electrons are turned back and the current ceases.

\[ \Delta V < 0: \text{A negative anode repels the electrons. Only the very fastest make it to the anode.} \]
Let the cathode be the point of zero potential energy.

Conserving energy as the photoelectron moves to the anode:

\[ K_f = K_i + e \Delta V \]

When the potential difference causes the very fastest electrons to have \( K_f = 0 \), this is the stopping potential:

\[ V_{\text{stop}} = \frac{K_{\text{max}}}{e} \]
In this experiment, a current is detected when ultraviolet light shines on the metal cathode. What happens to the current if the battery voltage is reduced to zero?

A. The current is unchanged.
B. The current decreases slightly.
C. The current becomes zero.
D. The current goes the other direction.
QuickCheck

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✅ B. The current decreases slightly.
Around 1900, Max Planck was able to predict the spectrum of blackbody radiation by assuming that atoms oscillating with frequency $f$ could only have specific energy values $E = 0, hf, 2hf, 3hf, \ldots$

- **Planck’s constant** is $h = 6.63 \times 10^{-34} \text{ J s}$
- In 1905, Albert Einstein suggested that the **electromagnetic radiation itself is quantized**!
- Einstein called each packet of energy a **light quantum**, with energy:

$$E = hf$$
Example 3 – The retina of your eye has three types of color photoreceptors, called cones, with maximum sensitivities at 437 nm, 533 nm, and 575 nm. For each, what is the energy of one quantum of light having that wavelength?
Einstein’s Explanation of the Photoelectric Effect

- Einstein was awarded the Nobel Prize not for his theory of relativity, but for his explanation of the photoelectric effect!
- Quanta of light energy were later given the name photons.
- Einstein framed three postulates about photons and their interaction with matter:
  1. Light of frequency $f$ consists of discrete quanta, each of energy $E = hf$. Each photon travels at the speed of light $c$.
  2. Photons are emitted or absorbed on an all-or-nothing basis. A substance can only emit or absorb an integer number of photons.
  3. A photon, when absorbed by a metal, delivers its entire energy to one electron.
An electron that has just absorbed a photon has energy $hf$.

If $hf$ is greater than or equal to the work function $E_0$, then the electron can escape from the metal.

The threshold frequency for ejection of photoelectrons is

$$f_0 = \frac{E_0}{h}$$
A more intense light means more photons of the same energy, not more energetic photons.

These photons eject a larger number of photoelectrons and cause a larger current, exactly as Lenard had observed.

Also, if each photon transfers its energy $hf$ to just one electron, that electron *immediately* has enough energy to escape.

The current should begin instantly, with no delay, exactly as Lenard had observed.
Einstein’s Explanation of the Photoelectric Effect

- There is a distribution of kinetic energies, because different photoelectrons require different amounts of energy to escape, but the *maximum* kinetic energy is

\[ K_{\text{max}} = E_{\text{elec}} - E_0 = hf - E_0 \]

- Einstein’s theory predicts that the stopping potential is related to light frequency by

\[ V_{\text{stop}} = \frac{K_{\text{max}}}{e} = \frac{hf - E_0}{e} \]

- The stopping potential does *not* depend on the intensity of the light, which is exactly what Lenard had observed.
Millikan measured the stopping potential as the light frequency was varied. A graph of his data and a fit to Einstein’s prediction is shown.

Millikan determined $h$ from this experiment, and found it to agree with Planck’s value determined in 1900:

$$V_{\text{stop}} = \frac{h}{e}(f - f_0)$$
QuickCheck

In an experiment to demonstrate the photoelectric effect, you shine a beam of monochromatic blue light on a metal plate. As a result, electrons are emitted by the plate. Complete the sentence: “If you increase the intensity of the light but keep the color of the light the same, the number of electrons emitted per second will _________ and the maximum kinetic energy of the emitted electrons will _________.

A. increase, increase
B. increase, stay the same
C. stay the same, increase
D. stay the same, stay the same
QuickCheck

What happens to this graph of the photoelectric current if the cathode’s work function is slightly increased?

D. None of these.
QuickCheck

What happens to this graph of the photoelectric current if the cathode’s work function is larger than the photon energy?

A.  
B.  
C.  
D. None of these.
Example 4 – What are the threshold frequencies and wavelengths for photoemission from sodium and from aluminum?
QuickCheck

What happens to this graph of the photoelectric current if the cathode’s work function is slightly increased?

A.  
B.  
C.  
D.  None of these.
Example 5 – For a particular cathode material in a photoelectric-effect experiment, you measure stopping potentials $V_0 = 1.0 \text{ V}$ for light of wavelength $\lambda = 600 \text{ nm}$, $2.0 \text{ V}$ for $400 \text{ nm}$, and $3.0 \text{ V}$ for $300 \text{ nm}$. Determine the work function $E_o$ for this material and the implied value of Planck’s constant $h$. 
In-class Activity #1 – While conducting a photoelectric-effect experiment with light of a certain frequency, you find that a reverse potential difference of 1.25 V is required to reduce the current to zero. Find the maximum kinetic energy and the maximum speed of the emitted photoelectrons.
Photon momentum

• Every particle that has energy must have momentum.

• Photons have zero rest mass, and a particle with zero rest mass and energy $E$ has momentum with magnitude $p$ given by $E = pc$.

• Thus the magnitude $p$ of the momentum of a photon is:

\[ p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda} \]

• The direction of the photon’s momentum is simply the direction in which the electromagnetic wave is moving.
Example 6 – A laser pointer with a power of 5.00 mW emits red light (650 nm). What is the magnitude of the momentum of each photon? Also, how many photons does the laser pointer emit each second?
**X-ray production**

- Shown is an experimental arrangement for making x rays.
- The next slide shows the resulting x-ray spectrum.
X-ray production

• The greater the kinetic energy of the electrons that strike the anode, the shorter the minimum wavelength of the x rays emitted by the anode.

• The photon model explains this behavior.

• Higher-energy electrons can convert their energy into higher-energy photons, which have a shorter wavelength.
QuickCheck

A beam of electrons is accelerated to high speed and aimed at a metal target. The electrons brake to a halt when they strike the target, and x-ray photons are produced. Complete the sentence: “If you increase the voltage used to accelerate the electrons, the x-ray photon energy will _________ and the x-ray photon wavelength will _________.

A. increase, increase  
B. increase, decrease  
C. decrease, increase  
D. decrease, decrease
Example 7 – Electrons in an X-ray tube accelerate through a potential difference of 10.0 kV before striking a target. If an electron produces one photon on impact with the target, what is the minimum wavelength of the resulting x rays? Find the answer by expressing energies in both SI units and electron volts.
Atomic electrons can absorb x rays.

Hence materials with many electrons per atom tend to be better x-ray absorbers than materials with few electrons.

Bones contain large amounts of elements such as phosphorus and calcium, with 15 and 20 electrons per atom, respectively.

In soft tissue, the predominant elements are hydrogen, carbon, and oxygen, with only 1, 6, and 8 electrons per atom, respectively.

Hence x rays are absorbed by bone but pass relatively easily through soft tissue.
X-ray scattering: The Compton experiment

• In the Compton experiment, x rays are scattered from electrons.

• The scattered x rays have a longer wavelength than the incident x rays, and the scattered wavelength depends on the scattering angle $\phi$. 
Compton scattering

- In **Compton scattering**, an incident photon collides with an electron that is initially at rest.
- The photon gives up part of its energy and momentum to the electron, which recoils as a result of this impact.
- The scattered photon flies off at an angle $\phi$ with respect to the incident direction, but it has less energy and less momentum than the incident photon.
- Therefore, the wavelength of the scattered photon $\lambda'$ is longer than the wavelength $\lambda$ of the incident photon.

\[
\lambda' - \lambda = \frac{h}{mc} (1 - \cos \phi)
\]

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Quick Check

When an x-ray photon bounces off an electron,

A. the photon wavelength decreases and the photon frequency decreases.

B. the photon wavelength decreases and the photon frequency increases.

C. the photon wavelength increases and the photon frequency decreases.

D. the photon wavelength increases and the photon frequency increases.
Example 8 – You use 0.124 nm x-ray photons in a Compton-scattering experiment. At what angle is the wavelength of the scattering x rays 1.0% longer than that of the incident x rays? At what angle is it 0.050% longer?
**Pair production**

- When gamma rays of sufficiently short wavelength are fired into a metal plate, they can convert into an electron and a **positron**, each of mass $m$ and rest energy $mc^2$.

- The photon model explains this: The photon wavelength must be so short that the photon energy is at least $2mc^2$. 

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Example 9 – An electron and a positron, initially far apart, move toward each other with the same speed. They collide head-on, annihilating each other and producing two photons. Find the energies, wavelengths, and frequencies of the photons if the initial kinetic energies of the electron and positron are both negligible and both 5.000 MeV. The electron rest energy is 0.511 MeV.